Plenary Lectures

A fast Multiscale method for the wave equation and its applications

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Numerical modeling of wave propagation in heterogeneous media is important in many inverse problems, which require repeated solves of the wave equation. Due to the complex nature, direct numerical simulations on the fine grid are prohibitively expensive. It is therefore important to develop efficient and accurate methods that allow the use of coarse grids. In this talk, we present a multiscale finite element method for wave propagation on a coarse grid. The proposed method is based on the Generalized Multiscale Finite Element Method (GMsFEM). To construct multiscale basis functions, we start with two snapshot spaces in each coarse-grid block where one represents the degrees of freedom on the boundary and the other represents the degrees of freedom in the interior. We use local spectral problems to identify important modes in each snapshot space. These local spectral problems are different from each other and their formulations are based on the analysis. To our best knowledge, this is the first time where multiple snapshot spaces and multiple spectral problems are used and necessary for efficient computations. Using the dominant modes from local spectral problems, multiscale basis functions are constructed to represent the solution space locally within each coarse block. These multiscale basis functions are coupled via the symmetric interior penalty discontinuous Galerkin method which provides a block diagonal mass matrix, and, consequently, results in fast computations in an explicit time discretization. Our methods' stability and spectral convergence are rigorously analyzed. Numerical examples are presented to show the performance of our methods. This is a joint work with Yalchin Efendiev and Wing Tat Leung. The research of Eric Chung is partially supported by Hong Kong RGC General Research Fund (Project: 400411).

Issues on Electro-Magnetic property mapping using Magnetic Resonance Imaging

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Recent research on probing electric and magnetic properties of tissue in-vivo using Magnetic Resonance Imaging (MRI) have seen an steep uprise due to its potential impact on health care management. However, reconstruction of the these properties both face unique inverse problems not yet readily solved. Here, we present the background and issues concerning this problem. Ad hoc solutions to overcome these issues are also presented.

Quantitative uniqueness for the Stokes system and its application.

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We study the local behavior of a solution to the Stokes system with singular coefficients in $\mathbb{R}^n$. One of our main results is a bound on the vanishing order of a nontrivial solution u satisfying the Stokes system, which is a quantitative version of the strong unique continuation property for u. Different from the previous known results, our strong unique continuation result only involves the velocity field u. Our proof relies on some delicate Carleman-type estimates. We first use these estimates to derive crucial optimal three-ball inequalities for u which is an important tool for stability estimates in inverse boundary value problems. Taking advantage of the optimality, we then derive an upper bound on the vanishing order of any nontrivial solution u to the Stokes system from those three-ball inequalities. As an application, we derive a minimal decaying rate at infinity of any nontrivial u satisfying the Stokes equation under some a priori assumptions.

Spatial encoding and decoding in magnetic resonance imaging: the forward and the inverse problems

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Magnetic resonance imaging (MRI) is a powerful diagnostic imaging tool in both clinical medicine and neuroscience because of its non-invasiveness, versatile contrasts, and high spatial resolution. In this talk, we will first briefly introduce the spatial encoding and decoding formalism of MRI and the associated imaging hardware. Then we will summarize recent technological advances in magnet, gradient coils, and radio-frequency coils. Preliminary results enabled by these hardware advances and the associated challenges and opportunities will also be presented. Lastly, we will discuss open questions related to how to further advance these technologies and how to optimally use these tools in order to achieve MRI with high spatiotemporal resolution.

Nondestructive evaluation techniques based on inverse analyses of temperature distribution data measured by infrared thermography

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In recent years, nondestructive evaluation (NDE) techniques based on temperature distribution data measured by infrared thermography has been getting an increasing attention, since performance of the infrared thermography has been drastically improved. The inverse analysis schemes to identify defect parameters such as defect shape and size

from temperature distribution data play important roles in the development of quantitative NDE techniques. In this presentation, applications of the inverse analysis method in the NDE techniques based on temperature distribution data measured by infrared thermography are shown, as well as practical applications such as the inspection of bridges, buildings and plants.

Inverse problem of acoustic-optics

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A method to reconstruct the optical properties of a highly-scattering medium from acousto-optic measurements is proposed. The method is based on the solution to an inverse problem for the radiative transport equation with internal data.

Image restoration: wavelet frame approach, total variation and beyond

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This talk is about the wavelet frame-based image and video restorations. We start with some of main ideas of wavelet frame based image restorations. Some of applications of wavelet frame based models image analysis and restorations will be shown. Examples of such applications include image and video inpainting, denoising, decomposition, image deblurring and blind deblurring, segmentation, CT image reconstruction, 3D reconstruction in electronmicroscopy, and etc. In all of these applications, spline wavelet frames derived from the unitary extension principle are used. This allows us to establish connections between wavelet frame base method and various PDE based methods, that include the total variation model, nonlinear diffusion PDE based methods, and model of Mumford-Shah. In fact, we will show that, when spline wavelet frames are used, all PDE based models can be understood by right chosen models of a wavelet frame method.

Acoustic scattering by a large number of small bodies and applications

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Let M be the number of bounded and Lipschitz regular obstacles $D_j, j := 1, ..., M$, having a maximum radius a, $a << 1$, located in a bounded domain Ω of $\mathbb{R}^3$. We are concerned with the acoustic scattering problem with a very large number of such obstacles, as $M := M(a) := O(a^{-s})$, $s \geq 0$, $a \to 0$, when they are arbitrarily distributed

in Ω with a minimum distance between them of the order $d := d(a) := O(a^t)$ with t in an appropriate range.

Our goal is to derive the limiting scattering problem as $a \to 0$. We will discuss the following results. There exist a threshold $M^* := O(a^{s^*})$, which depends on the type of scatterers we have (for instance: Dirichlet, Neumann or Impedance) such that :

1. if $s < s^*$, then the collection of small bodies has no effect on the background. The scattered fields tend to vanish.
2. if $s = s^*$, then the collection of small bodies behaves as an acoustic medium modeled by an index of refraction characterized by two coefficients: one related to the geometry of the bodies (i.e. capacitance or perimeter) and the other one to their local distribution.
3. if $s > s^*$, then the collection of small bodies behaves as an impenetrable extended scatterer, i.e. the limiting scattering problem behaves as the exterior of an extended obstacle with Dirichlet boundary condition on its surface.

We assume no periodicity in the distribution of the small bodies. In addition, we provide error estimates in terms of a. Some applications of the results in (2) and (3) will be discussed.

Geometry and Calderón problems

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The problem of determining the electrical conductivity of a body by making voltage and current measurements on the object's surface has various applications in fields such as oil exploration and early detection of malignant breast tumour. This classical problem posed by Calderón remained open until the late '80s when it was finally solved in a breakthrough paper by Sylvester-Uhlmann.

In the recent years, geometry has played an important role in this problem. The unexpected connection of this subject to fields such as dynamical systems, symplectic geometry, and Riemannian geometry has led to some interesting progress. This talk will be an overview of some of the recent results and an outline of the techniques used to treat this problem.

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Seeing through space time

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We consider inverse problems for the Einstein equation with a time-depending metric on a 4-dimensional globally hyperbolic Lorentzian manifold. We formulate the concept of active measurements for relativistic models. We do this by coupling Einstein equations with equations for scalar fields.

The inverse problem we study is the question, do the observations of the solutions of the coupled system in an open subset U of the space-time with the sources supported in U determine the properties of the metric in a larger domain? To study this problem we define the concept of light observation sets and show that these sets determine the conformal class of the metric. This corresponds to passive observations from a distant area of space which is filled by light sources. We will also consider inverse problems for other non-linear hyperbolic equations.

This is joint work with Y. Kurylev and M. Lassas

Inverse problem for a phase field system by measurements of one component

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We consider an inverse problem for a linearized phase field system and discuss the determination of two coefficients, that is, a thermal conductivity and a mobility, by measuring only one component in a small subdomain. We prove Lipschitz stability for this inverse problem in terms of Bukhgeim-Klibanov method. The proof is based upon a Carleman estimate by extra data of one component.

Inverse scattering problems for complex obstacles

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The theory of inverse obstacle scattering has been an active research filed in applied mathematics for a long time. The aim of research in this filed is to identify unknown objects through the use of acoustic, electromagnetic or elastic waves. In this talk, I will present our recent works on inverse scattering problems for complex obstacles. First, we consider inverse scattering problems for obstacles with impedance boundary conditions. It is found that impedance boundary conditions could be used to approximately describe some scattering problems involving complex geometric structures or physical properties. In this case, our task is to reconstruct both the shape of an unknown obstacle and the boundary impedance from the measured data of scattered fields. Second, we are concerned with inverse scattering problems with oblique derivative boundary conditions, which arises in some new scattering models such as the scattering of tidal waves by islands under suitable assumptions. The uniqueness issue and reconstruction scheme are investigated. The talk is based on my joint works with Prof. Jijun Liu (Southeast University) and Prof. Gen Nakamura (Inha University).

Inverse source problems for time-fractional diffusion equations

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In this talk, we consider two inverse source problems for time-fractional diffusion equations. That is to determine a space-dependent source term or a time-dependant source term in the time-fractional diffusion equations from additional noisy data. Based on the series expression of the solutions, the original inverse problems can be transformed into the first kind integral equations. The uniqueness and conditional stability for the source terms will be investigated. Further, we propose two regularization methods to deal with the inverse source problems. Numerical examples are provided to show the effectiveness of the proposed methods.

Patch-based low-rank and sparsity penalty for biomedical imaging applications

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In MRI, CT, and PET, to retain sufficient temporal resolution, the number of samples or its signal to noise ratio are often not sufficient enough, so conventional reconstruction algorithms produce images with artifacts. To address this problem, many advanced reconstruction algorithms have been developed using various spatio-temporal regularizations. The main goal of this work is to develop a novel spatio-temporal regularization approach that exploits inherent similarities within and across frames. One of the main contributions of this paper is to demonstrate that such correlations can be exploited using a low rank constraint of patches in spatial-, temporal- or data domain. Using simulation results and real in vivo experiments with MRI, CT, and PET, we confirm that the proposed algorithm can improve image quality and extract high quality parameters.